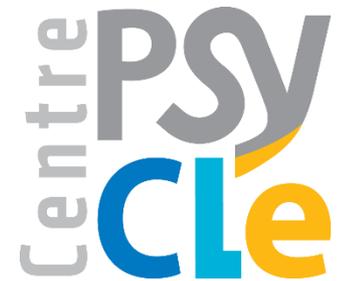




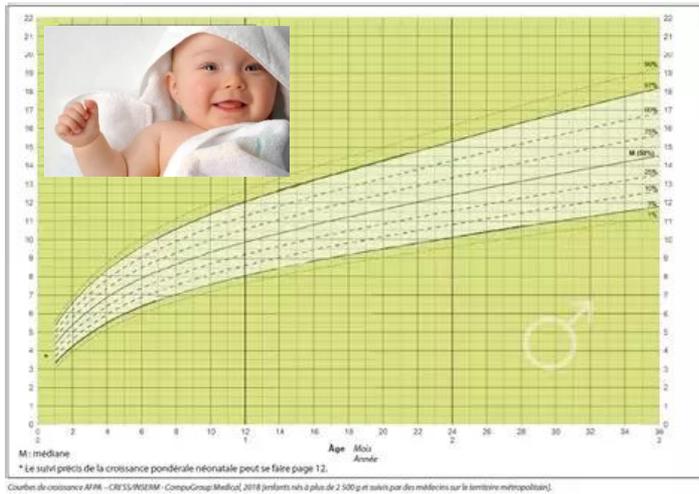
8^e ATELIER MODEVAIIA – 7 -9 Juin 2023

**SINGULAR SPECTRUM ANALYSIS (SSA)
TENSOR DECOMPOSITION (PARAFAC)
TENSOR ON TENSOR REGRESSION
ET PLUS SI AFFINITÉ**

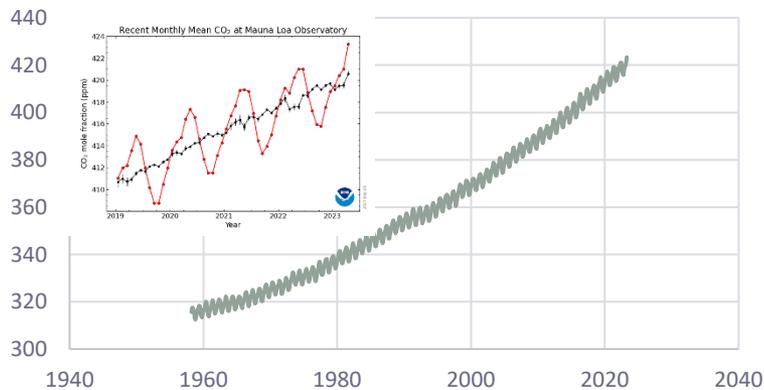
Dauvier, B.



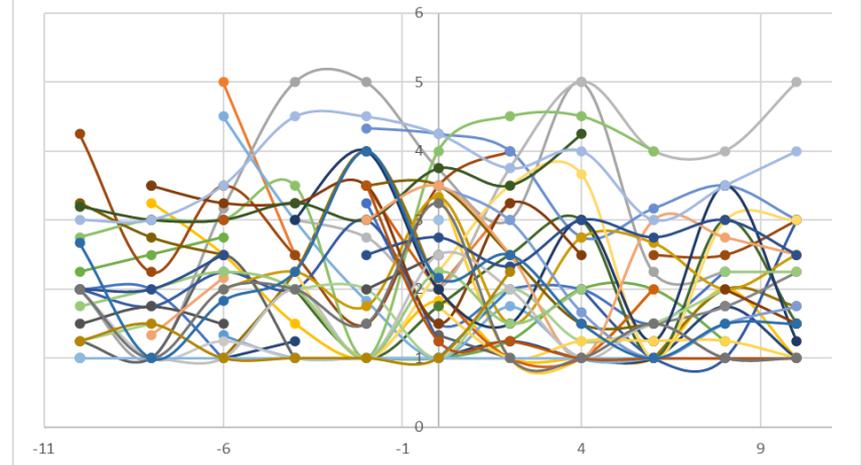
Modèles de croissance et/ou cyclique? À effets mixtes (inter et intra)?



Mauna Loa CO2



Rumination pendant le cycle féminin

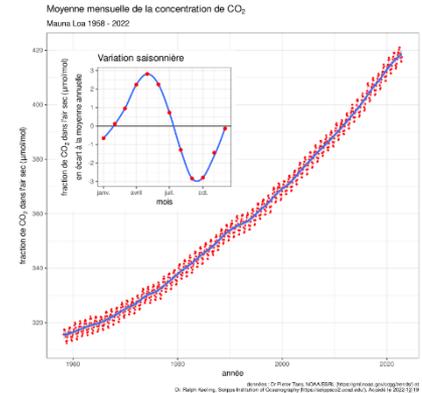


Singular spectrum analysis (SSA, Librairie Rssa)

Analyse de série chronologique univariée ou multivariée

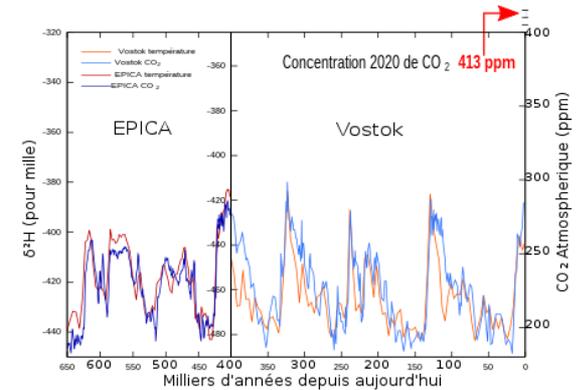
Données CO2 Mauna Loa

La courbe de Keeling est un graphique de l'évolution de la concentration de dioxyde de carbone (CO2) dans l'atmosphère terrestre depuis 1958. Elle est basée sur les mesures en continu faites au Mauna Loa Observatory à Hawaï.



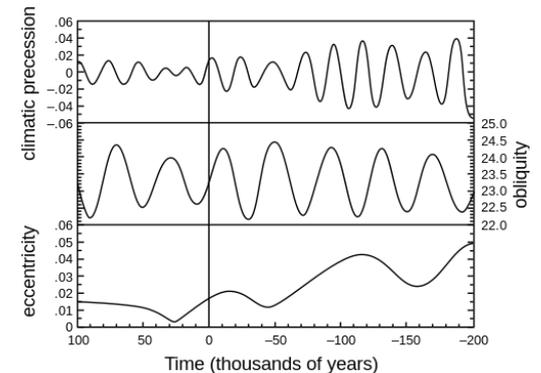
Données EPICA

Le projet EPICA (pour European Project for Ice Coring in Antarctica) est un projet européen de forage dans les glaces profondes de l'Antarctique. Température et CO2.



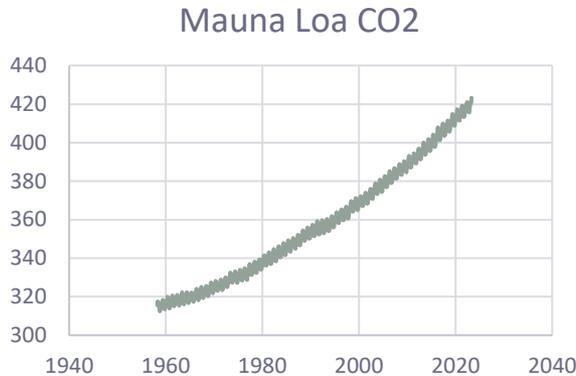
Cycles de Milankovitch

Les cycles de Milankovitch sont des variations cycliques de paramètres de l'orbite de la Terre (excentricité, obliquité, precession) qui engendrent des variations du climat terrestre.



Etape 1: Embedding

Agencement de la série chronologique dans une matrice de trajectoire



$$\mathbf{X} = (\mathbf{x}_1 | \dots | \mathbf{x}_N) = \begin{pmatrix} x_1 & x_2 & x_3 & \dots & x_N \\ x_2 & x_3 & x_4 & \dots & x_{N+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_L & x_{L+1} & x_{L+2} & \dots & x_T \end{pmatrix}$$

Matrice de Hankel

$$\begin{pmatrix} a & b & c & d & e \\ b & c & d & e & f \\ c & d & e & f & g \\ d & e & f & g & h \\ e & f & g & h & i \end{pmatrix}$$

Matrice de Toeplitz

$$\begin{pmatrix} a & b & c & d & e \\ f & a & b & c & d \\ g & f & a & b & c \\ h & g & f & a & b \\ i & h & g & f & a \end{pmatrix}.$$

Nécessité de définir la fenêtre temporelle, i.e. le nombre de colonnes.
Moitié de la série par défaut dans `ssa()`.

Etape 2: Décomposition en valeurs singulières (SVD) de la matrice de trajectoire -> ACP

$$A = \sum_{i=1}^n \sigma_i u_i v_i^T$$

$$A = U D V^T$$

← Left singular vectors
↑ Singular values
← Right singular vectors

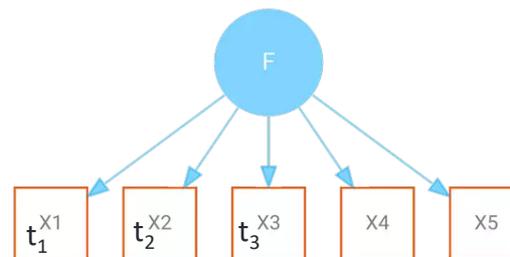
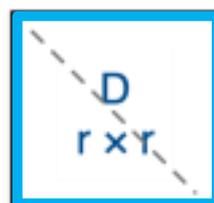
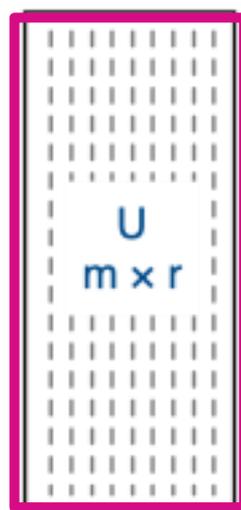
Scores factoriels

Valeurs Propres

Saturations

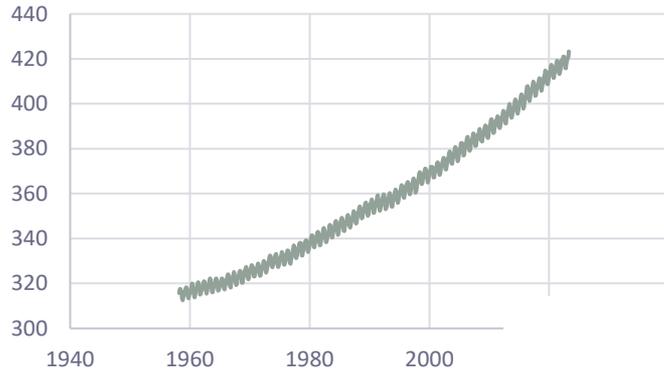


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Etape 3: Regroupement des composantes

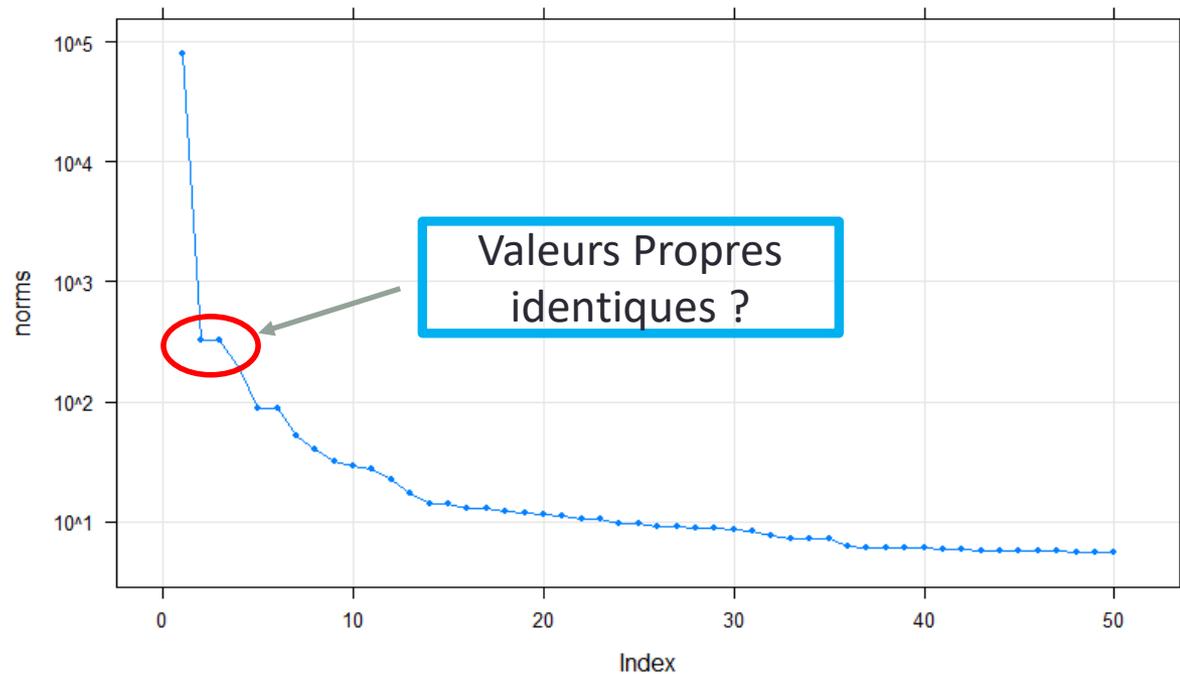
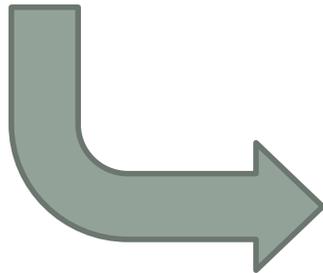
Mauna Loa CO2



Graphique des Valeurs Propres

Component norms

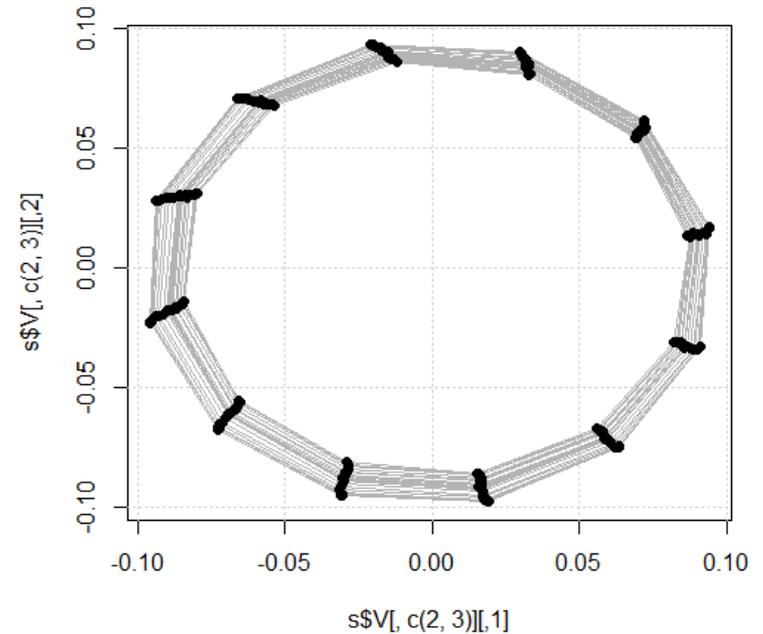
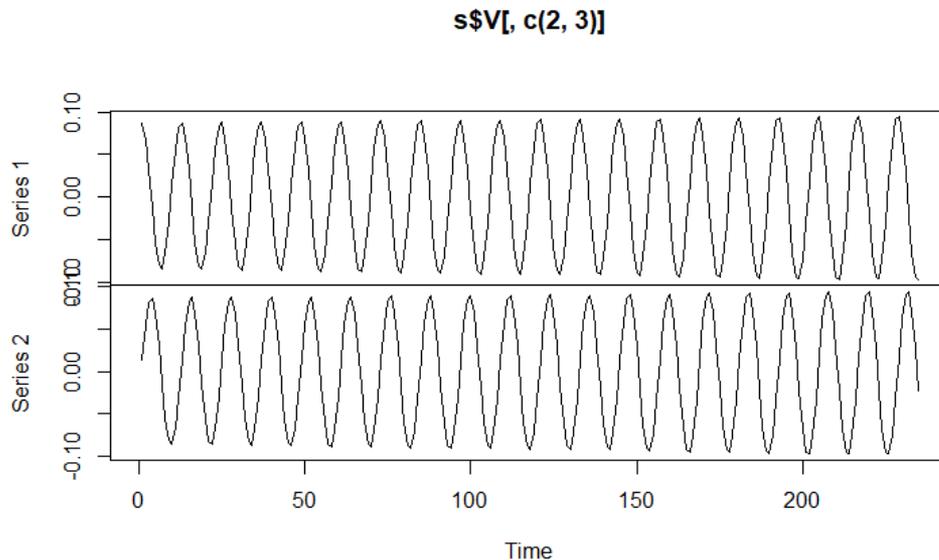
SVD



```
s <- ssa(co2)  
plot(s)
```

Etape 3: Regroupement des composantes

Valeurs Propres identiques pour les composantes 2 et 3?



Les « saturations » au cours du temps présentent des oscillations de même fréquence. Elles captent le même phénomène cyclique.

-> il est possible de les regrouper

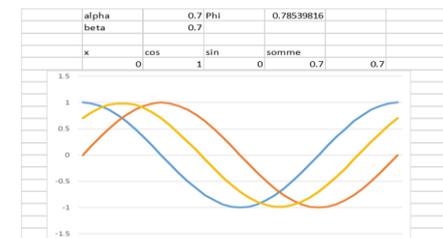
```
plot.ts(s$V[,c(2,3)])
```

Ajuster une sinusoïde avec un modèle linéaire

$$\alpha \sin x + \beta \cos x = \sqrt{\alpha^2 + \beta^2} \sin(x + \varphi)$$

où

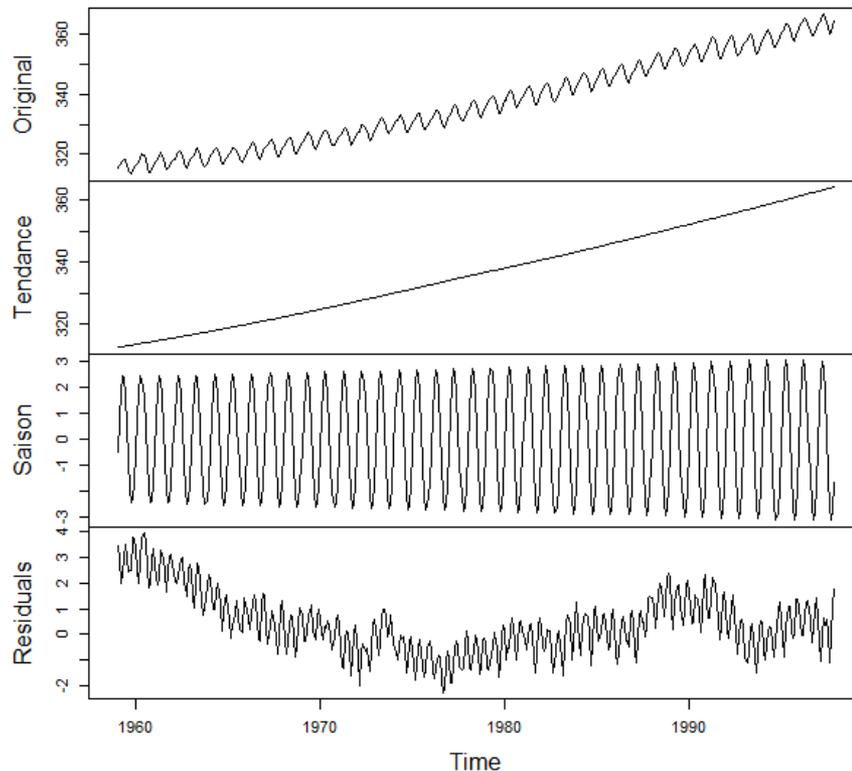
$$\varphi = \arctan(\beta/\alpha) \text{ si } \alpha \text{ est positif et } \varphi = \arctan(\beta/\alpha) + \pi \text{ sinon.}$$



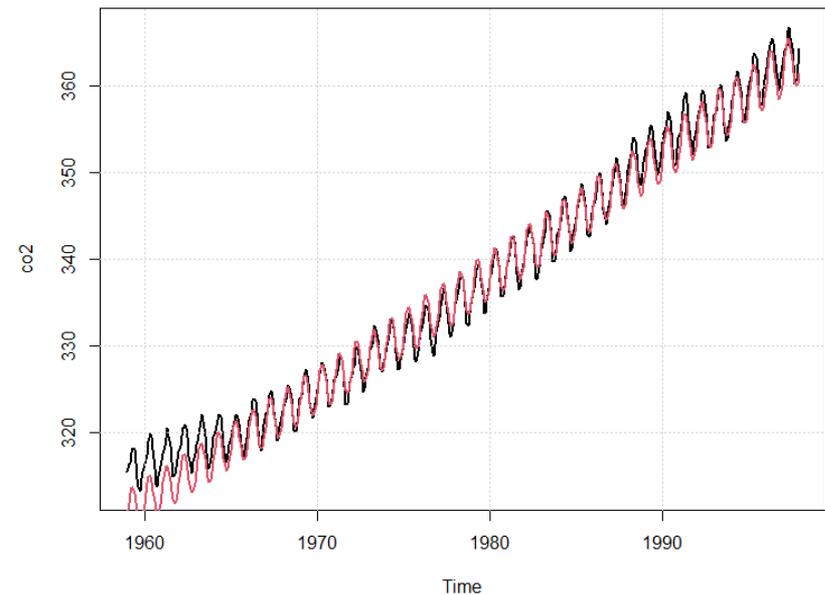
Etape 4: Reconstruction

3 composantes

Reconstructed Series



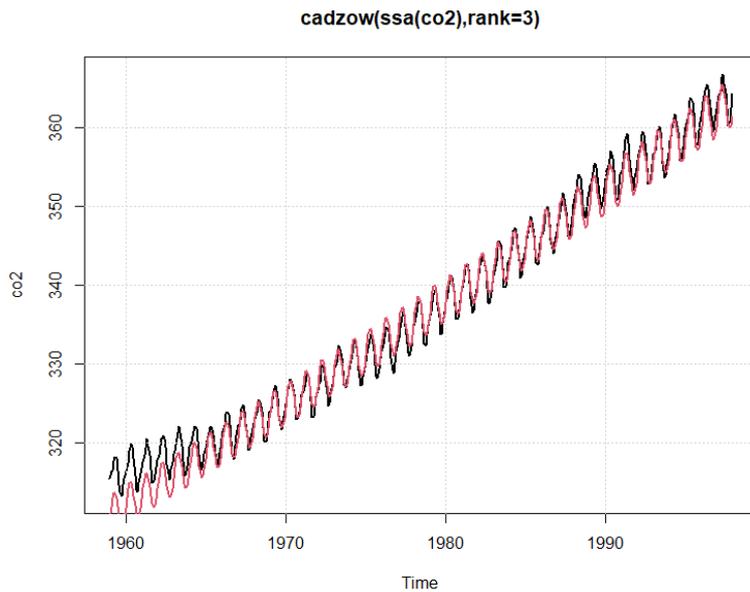
cadzow(ssa(co2),rank=3)



```
r <- reconstruct(s, groups = list(Tendance =c(1),Saison=c(2, 3)))  
plot(r)
```

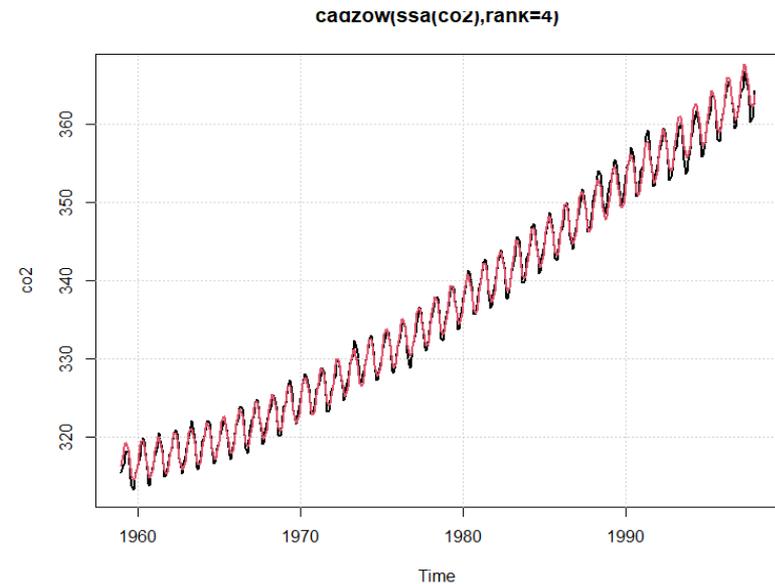
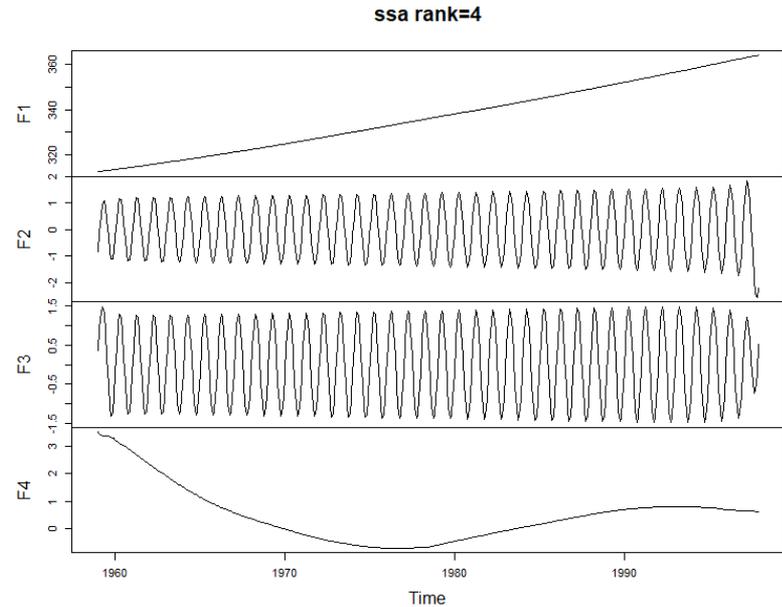
Etape 4: Reconstruction

3 composantes



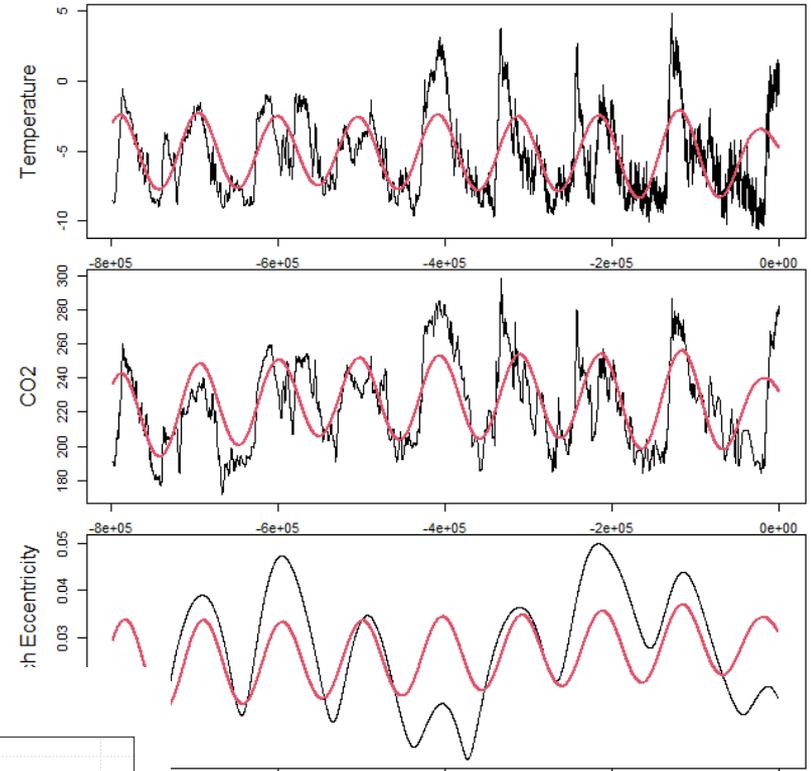
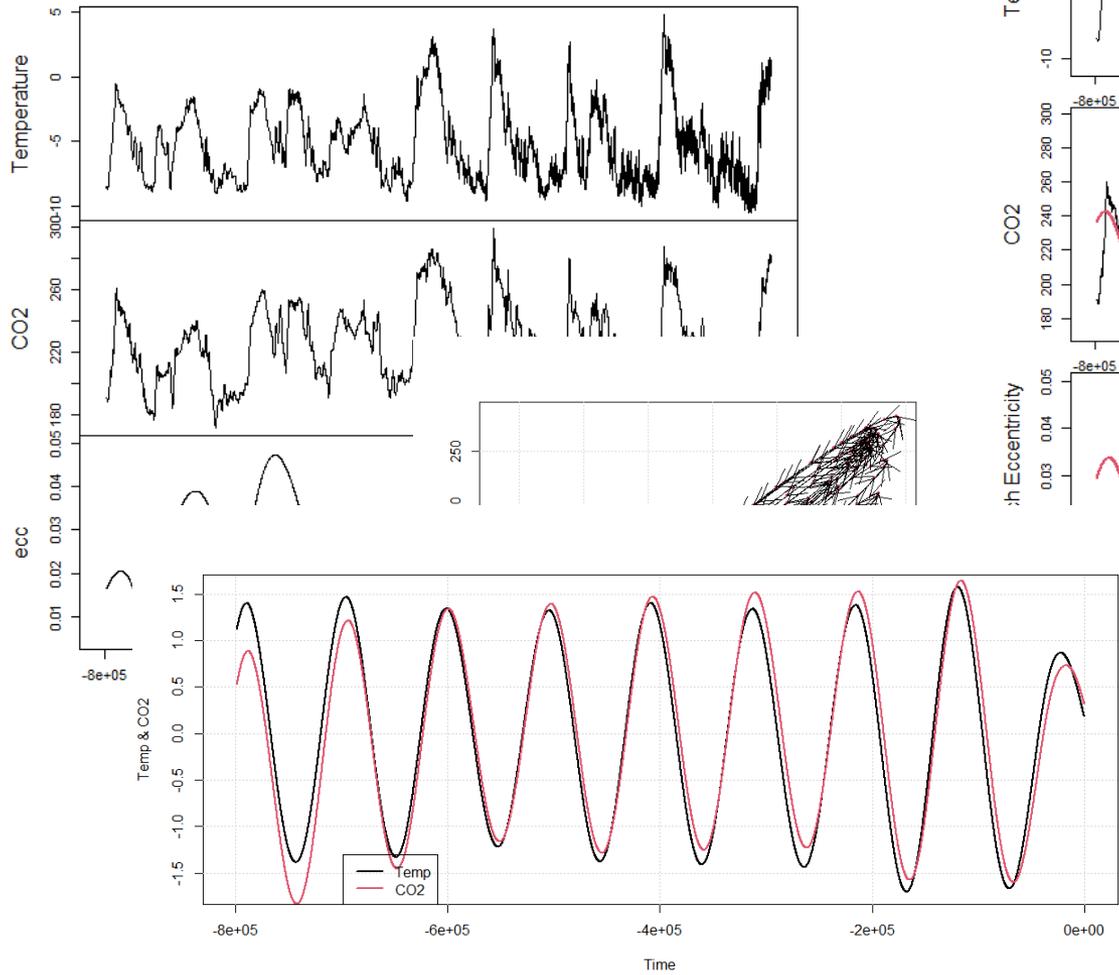
```
cad=cadzow(ssa(co2),rank=3)  
plot(cad)
```

4 composantes



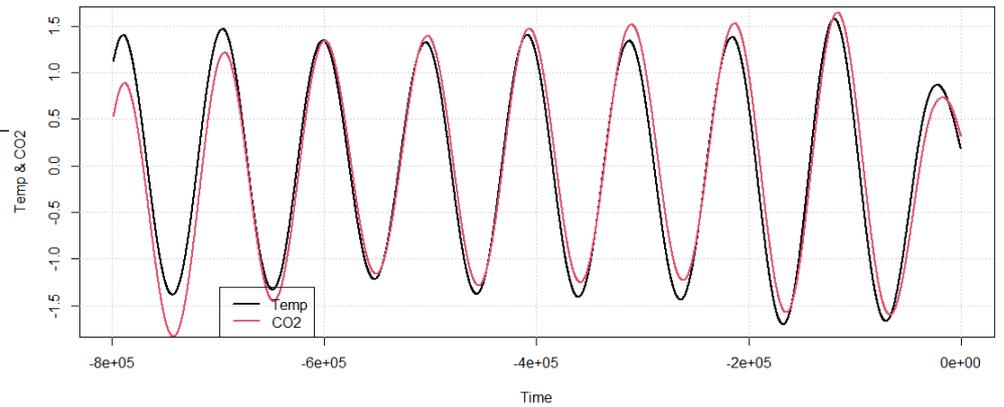
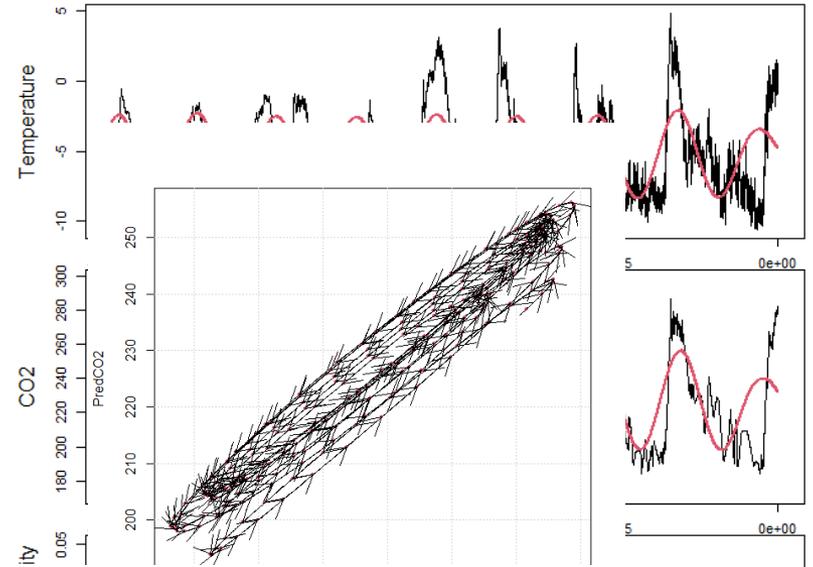
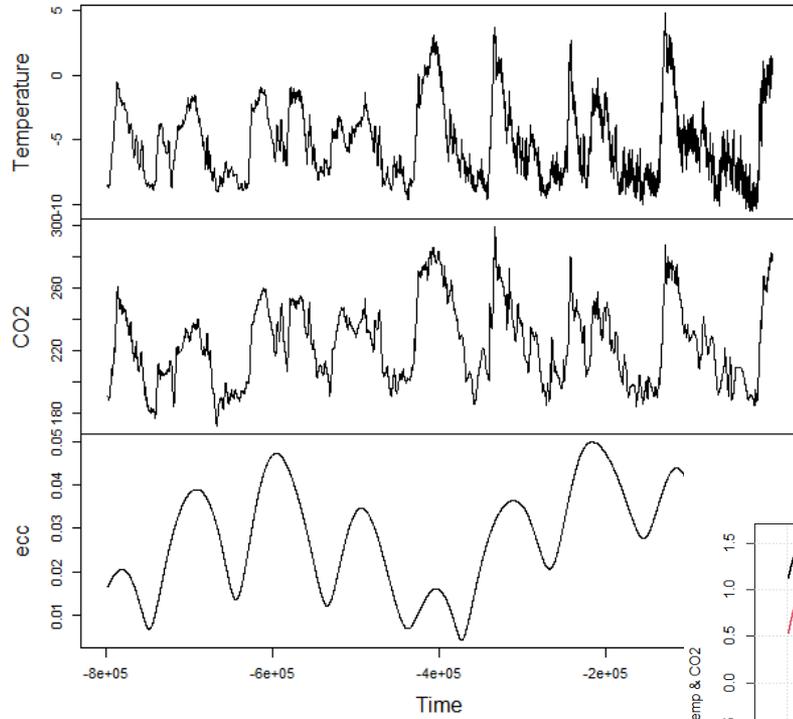
Aaaa

Epica data European Project for Ice Coring in Antarctica

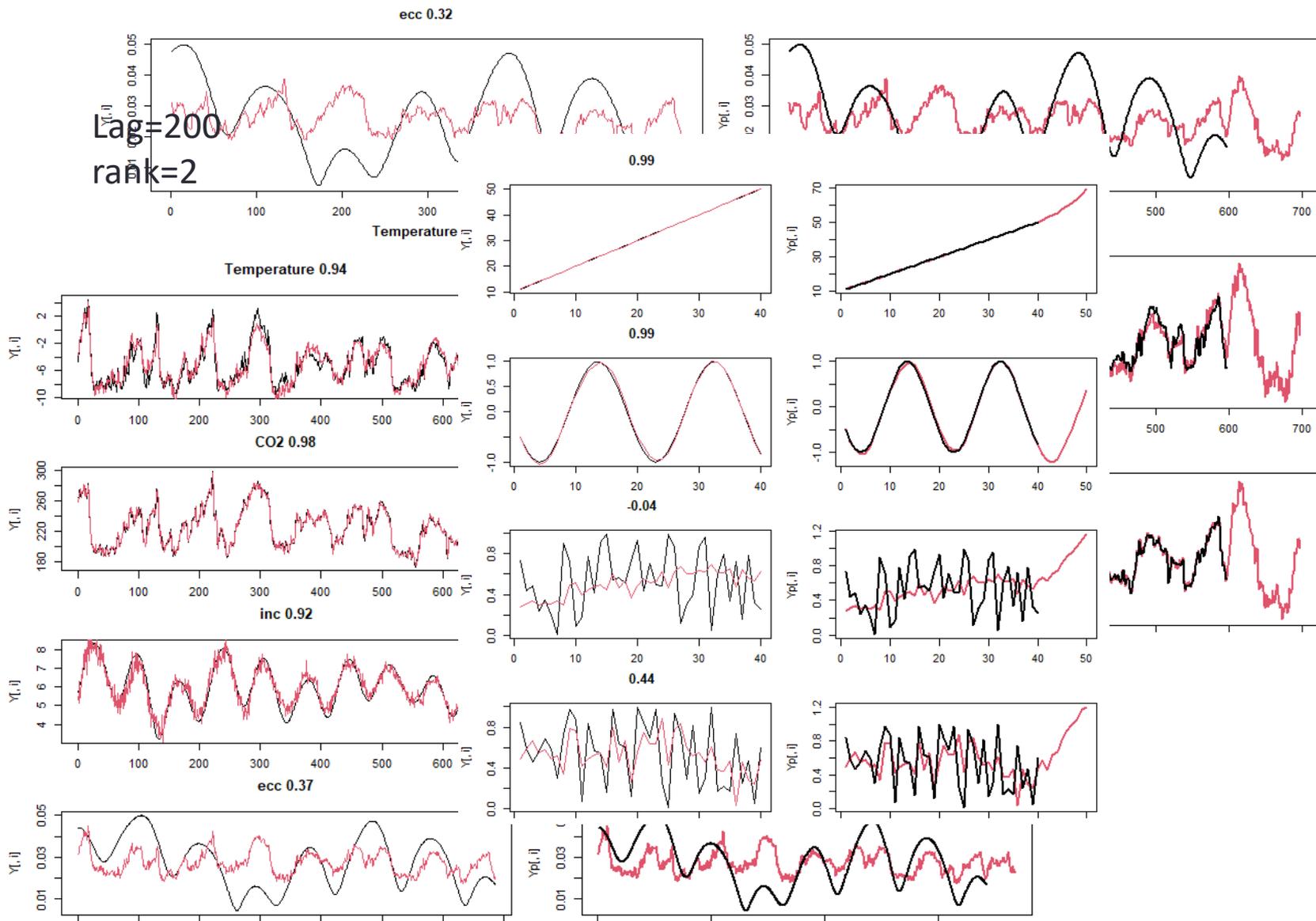


Aaaa

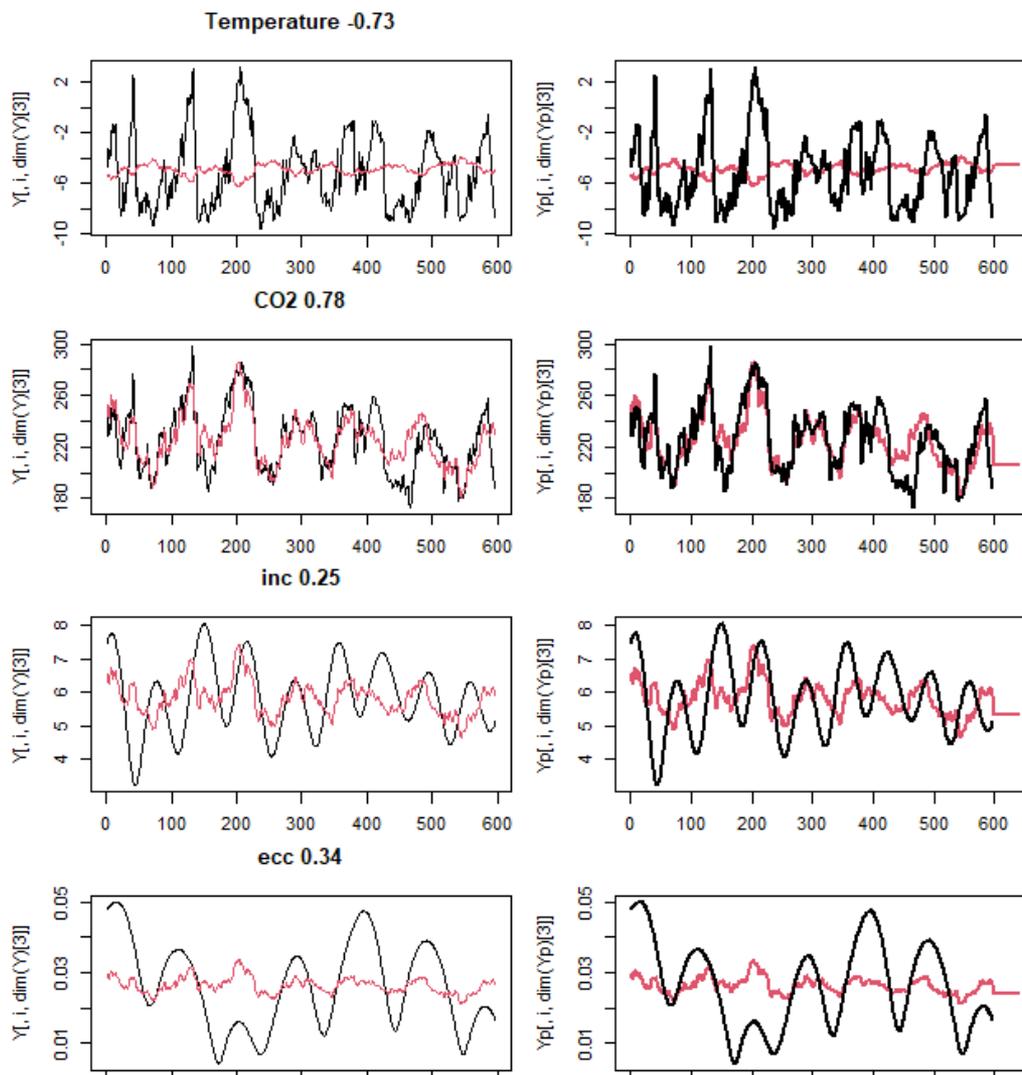
Epica data European Project for Ice Coring in Antarctica

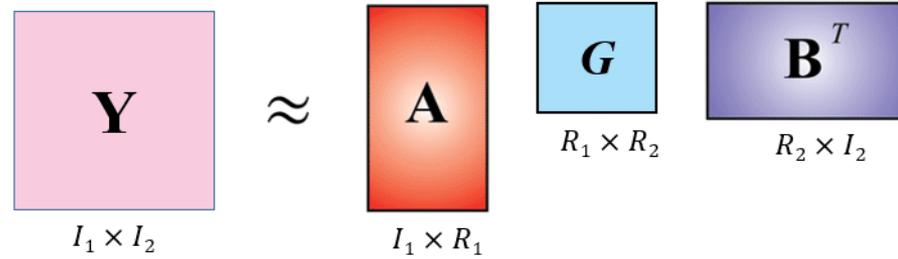


Aaaa

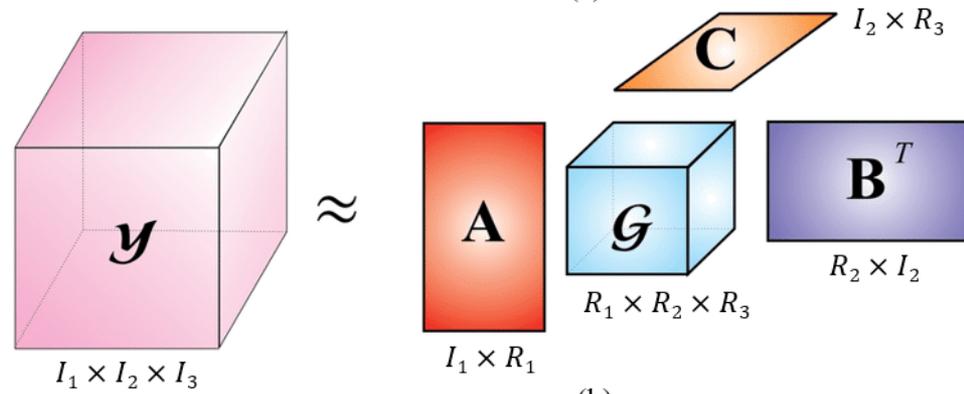


Aaaa





(a)



(b)